Numeric Response Questions

Circles

Q.1 Find the area of square inscribed in a circle $x^2 + y^2 - 6x - 8y = 0$.

Q.2 If the circumference of the circle $x^2 + y^2 + 8x + 8y - b = 0$ is bisected by the circle $x^2 + y^2 - 2x + 4y + a = 0$ then find **a + b**.

Q.3 Tangents are drawn from point (4,5) to the circle $x^2 + y^2 - 4x - 2y - 11 = 0$. Find the area of the quadrilateral formed by these tangents and radii joining their point of contacte.

Q.4 Find the sum of square of the length of the chords intercepted by the line $x + y = n; n \in N$ on the circle $x^2 + y^2 = 4$

Q.5 Two circle of equal radius r cut orthogonally. If their centres are (2,3) and (5,6) then find value of r.

Q.6 Find the number of common tangent(s) to the circles $x^2 + y^2 + 2x + 8y - 23 = 0$ and $x^2 + y^2 - 4x - 10y + 19 = 0$

Q.7 If the circle $x^2 + y^2 + 4x + 22y + c = 0$ bisects the circumferenoe of the circle $x^2 + y^2 - 2x + 8y - d = 0$, then find c + d

Q.8 Find the greatest distance of the point $P(10,7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$,

Q.9 If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then find the length of PQ.

Q.10 If the length of the common chord of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$ is $\frac{k}{\sqrt{2}}$ then find k.

Q.11 Find maximum number of circles possible touching both axes and line $3x + 4y = 12$.

Q.12 If the radius of the circle passing through the point (6,2), two of whose diameters are $x + y = 6$ and $x + 2y = 4$ is $5k$ then find k.

Q.13 If the equation $\lambda x^2 + (2\lambda - 3)y^2 - 4x - 1 = 0$ represents a circle whose radius is $\frac{\sqrt{7}}{k}$ then find k.

Q.14 If $(-3,2)$ lies on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which is concentric with the circle $x^2 + y^2 + 6x + 8y - 5 = 0$, then find value of c

Q.15 If the line $y = 7x - 25$ meets the circle $x^2 + y^4 = 25$ in the points A, B, and the distance between A and B is $k\sqrt{2}$ then find k.

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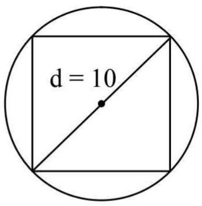
ANSWER KEY

1. 50.00	**2.** –56.00	**3.** 8.00	**4.** 22.00	**5.** 3.00	**6.** 3.00	**7.** 50.00
8. 15.00	**9.** 5.00	**10.** 4.00	**11.** 4.00	**12.** 2.00	**13.** 3.00	**14.** –11.00
15. 5.00						

Hints & Solutions

1.



Circle $x^2 + y^2 - 6x - 8y = 0$

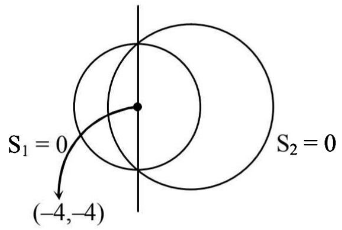
$r = \sqrt{9+16} = 5$

diameter = 10

Area of square $= \frac{1}{2}(\text{diagonal})^2$

Area $= \frac{1}{2}(10)^2 = 50$

2. Common chord is longest



Equation of common chord

$S_1 - S_2 = 0$

$10x + 4y - b - a = 0$

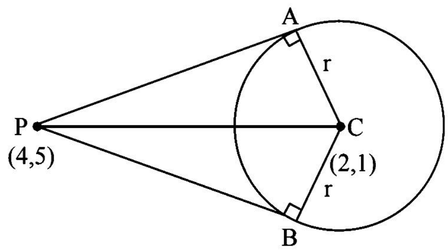
it passes through

$\therefore (-4, -4)$

$-40 - 16 = a + b$

$\Rightarrow a + b = -56$

3.



Area of $\square$ PACB = 2(Area of ΔPAC)

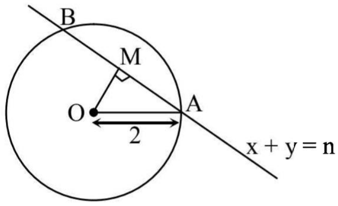
$= 2.\frac{1}{2}.r.PA$

$= r.PA$

$= r\sqrt{S_1}$

$= 4\sqrt{16+25-16-10-11} = 4{\cdot}2 = 8$

4.



AB = 2 AM

$AB^2 = 4\ AM^2$

$4\left(4 - \frac{n^2}{2}\right) = 2\,(8 - n^2),\ n \in N$

n = 1 or n = 2 ($\because$ length should be + ve)

Hence required sum

$= 2\,(8 - 1^2 + 8 - 2^2)$

$= 2 \times 11 = 22$

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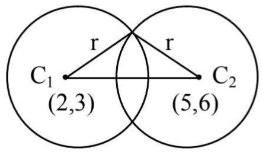
5.



orthogonal intersection

$r_1^2 + r_2^2 = d^2$

$r^2 + r^2 = (5 - 2)^2 + (6 - 3)^2$

$2r^2 = 9 + 9$

$r^2 = \dfrac{18}{2}$

$\Rightarrow r^2 = 9 \Rightarrow r = 3$

6. $C_1 = (-1, -4)$; $C_2 = (2, 5)$;

$r_1 = \sqrt{1+16+23} = 2\sqrt{10}$;

$r_2 = \sqrt{4+25+19} = \sqrt{10}$;

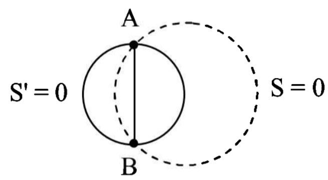
$C_1C_2 = \sqrt{9+18} = 3\sqrt{10}$

$\Rightarrow C_1C_2 = r_1 + r_2$

Hence, circles touch externally.

7. Equation of common chord $\boxed{S - S' = 0}$

$6x + 14y + c + d = 0$



It passes through centre (1, – 4)

$\therefore\ 6 - 56 + c + d = 0$

$c + d = 50$

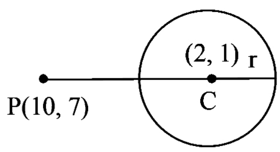
8. $C(2, 1)$; $r = \sqrt{4+1+20} = 5$

Greatest distance = PC + r

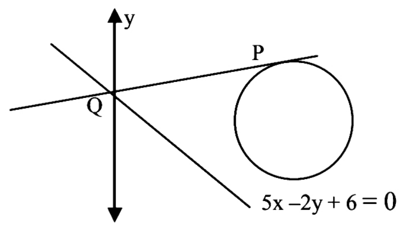
$= \sqrt{(10-2)^2 + (7-1)^2} + 5$

$= 10 + 5$

$= 15$



9.



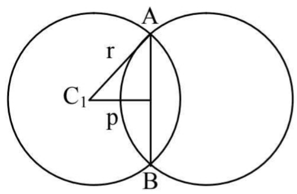
line $5x - 2y + 6 = 0$ intersects y-axis at Q (0, 3)

length $PQ = \sqrt{51}$

$= \sqrt{25}$

$= 5$

10. equation of common chord



$S - S' = 0$

$2x + 1 = 0$

$C_1\left(-1, -\dfrac{3}{2}\right)$,

$r_1 = \sqrt{1 + \dfrac{9}{4} - 1} = \sqrt{\dfrac{9}{4}}$

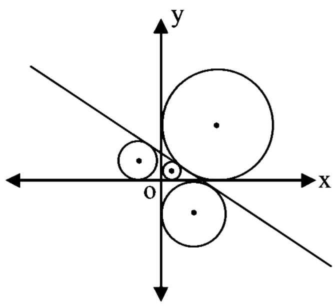
$p = \left|\dfrac{2(-1)+1}{\sqrt{4}}\right| = \dfrac{1}{2}$

$AB = 2\sqrt{r^2 - p^2}$

$AB = 2\sqrt{\dfrac{9}{4} - \dfrac{1}{4}}$

$= 2\sqrt{2}$

11. Hint : draw fig.



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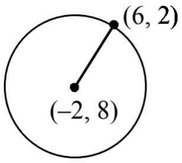
12. Diameter $x + y = 6$

and $x + 2y = 4$

$\therefore$ centre $(-2, 8)$



$r = \sqrt{(6+2)^2 + (2-8)^2} = 10$

13. Given equation is

$\lambda x^2 + (2\lambda - 3) y^2 - 4x - 1 = 0$

Here $a = \lambda$, $b = (2\lambda - 3)$

if represent a circle, if $a = b$

$\lambda = 2\lambda - 3$

$\lambda = 3$

$h = 0$

The equation becomes

$3x^2 + 3y^2 - 4x - 1 = 0$

$x^2 + y^2 - \frac{4}{3}x - \frac{1}{3} = 0$

here $g = -\frac{2}{3}$, $c = -\frac{1}{3}$, $f = 0$

$\therefore$ radius $= \sqrt{\left(-\frac{2}{3}\right)^2 + (-)\left(-\frac{1}{3}\right)}$

$= \sqrt{\frac{4}{9} + \frac{1}{3}} = \frac{\sqrt{7}}{3}$

14. $(-3, 2)$ lies on the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

and cancentric with the circle

$x^2 + y^2 + 6x + 8y - 5 = 0$

$\therefore (-3)^2 + (2)^2 + 2(3)(-3) + 2(4)(2) + c = 0$

$\Rightarrow c = -11$

15. Intersection point of line $y = 7x - 25$ and circle $x^2 + y^2 = 25$ is $x^2 + (7x - 25)^2 = 25$

$\Rightarrow 50x^2 - 350x + 600 = 0$

$x = 3, 4 \Rightarrow y = -4, 3$

so $A = (3, -4)$, $B = (4, 3)$

so $AB = \sqrt{(4-3)^2 + (3+4)^2} = 5\sqrt{2}$

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